

# Double Phase Transitions of Kerr-AdS Black Hole

Yu Dai Tsai

NTHU PHYS

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- Your works here? contact: s9822703@nthu.edu.tw

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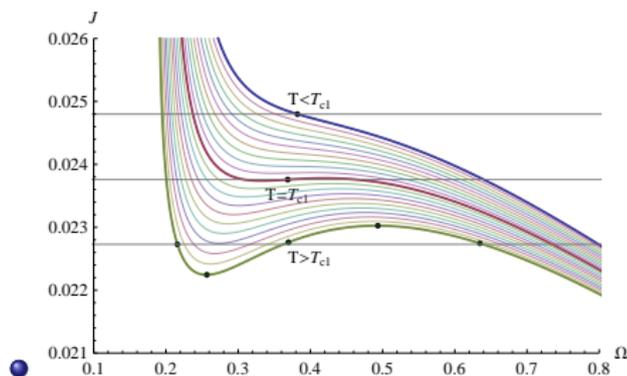


Figure: The critical isotherms near the critical temperature  $T_{c1}$ .

The Kerr-AdS metric

$$ds^2 = -\frac{\Delta_r}{\Sigma} \left( dt - \frac{a \sin^2 \theta}{\Xi} d\phi \right)^2 + \frac{\Sigma}{\Delta_r} dr^2 + \frac{\Sigma}{\Delta_\theta} d\theta^2 + \frac{\Delta_\theta \sin^2 \theta}{\Sigma} \left( a dt - \frac{r^2 + a^2}{\Xi} d\phi \right)^2 \quad (1)$$

where

$$\Delta_r = (r^2 + a^2) \left( 1 + \frac{r^2}{l^2} \right) - 2Mr, \quad \Xi = 1 - \frac{a^2}{l^2},$$
$$\Delta_\theta = 1 - \frac{a^2}{l^2} \cos^2 \theta, \quad \Sigma = r^2 + a^2 \cos^2 \theta. \quad (2)$$

Gibbons et al. in arXiv:hep-th/0408217v3 (2006)

The “physical” mass (or energy)  $E$  and angular momentum  $J$  is defined as

$$E = \frac{M}{\Xi^2}, \quad J = \frac{Ma}{\Xi^2}. \quad (3)$$

The Hawking temperature and the entropy  $T = \frac{\kappa}{2\pi}$  and  $S = \frac{A}{4}$ . The angular velocity of horizon  $\Omega$ ,

$$\Omega = -\frac{g_{t\phi}}{g_{\phi\phi}} = \frac{a\Xi[(r^2 + a^2)\Delta_\theta - \Delta_r]}{\Gamma}, \quad \Delta_r \rightarrow 0, \quad (4)$$

where  $\Gamma = (r^2 + a^2)^2 \Delta_\theta - \Delta_r a^2 \sin^2 \theta$ .

The relation between each quantity gives us the first law of the black hole thermodynamics, which is

$$dE = TdS + \Omega\delta J . \quad (5)$$

# The van der Waals system

a: attraction between each particle, b: the volume of the particle

$$\left(p + \frac{n^2 a}{V^2}\right)(V - nb) = nRT$$

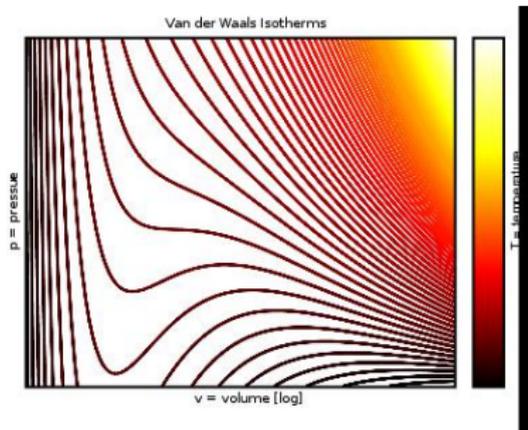


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# The Kerr-AdS black hole

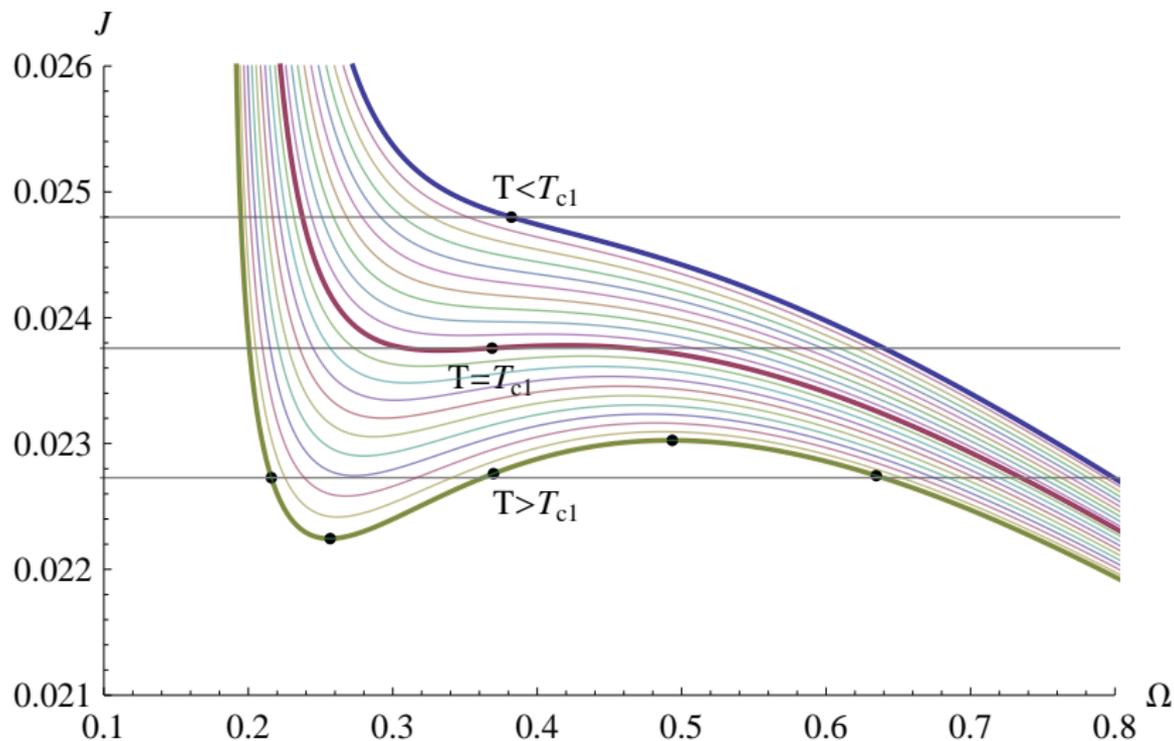


Figure: The critical isotherms near the critical temperature  $T_{cl}$ .

critical isotherm:

$$\Omega - \Omega_c = A_\delta |J - J_c|^\delta \text{sign}(J - J_c), \quad (T = T_c). \quad (6)$$

order parameter:

$$\eta_1 = -A_\beta (T - T_c)^\beta, \quad (T > T_c). \quad (7)$$

heat capacity

$$C_J = \begin{cases} A_{\alpha'} \{-(T - T_c)\}^{-\alpha'}, & (T < T_c) \\ A_{\alpha'} \{+(T - T_c)\}^{-\alpha}, & (T > T_c) \end{cases}. \quad (8)$$

isothermal compressibility

$$\kappa_T = \begin{cases} A_{\gamma'} \{-(T - T_c)\}^{-\gamma'}, & (T < T_c) \\ A_\gamma \{+(T - T_c)\}^{-\gamma}, & (T > T_c) \end{cases}. \quad (9)$$

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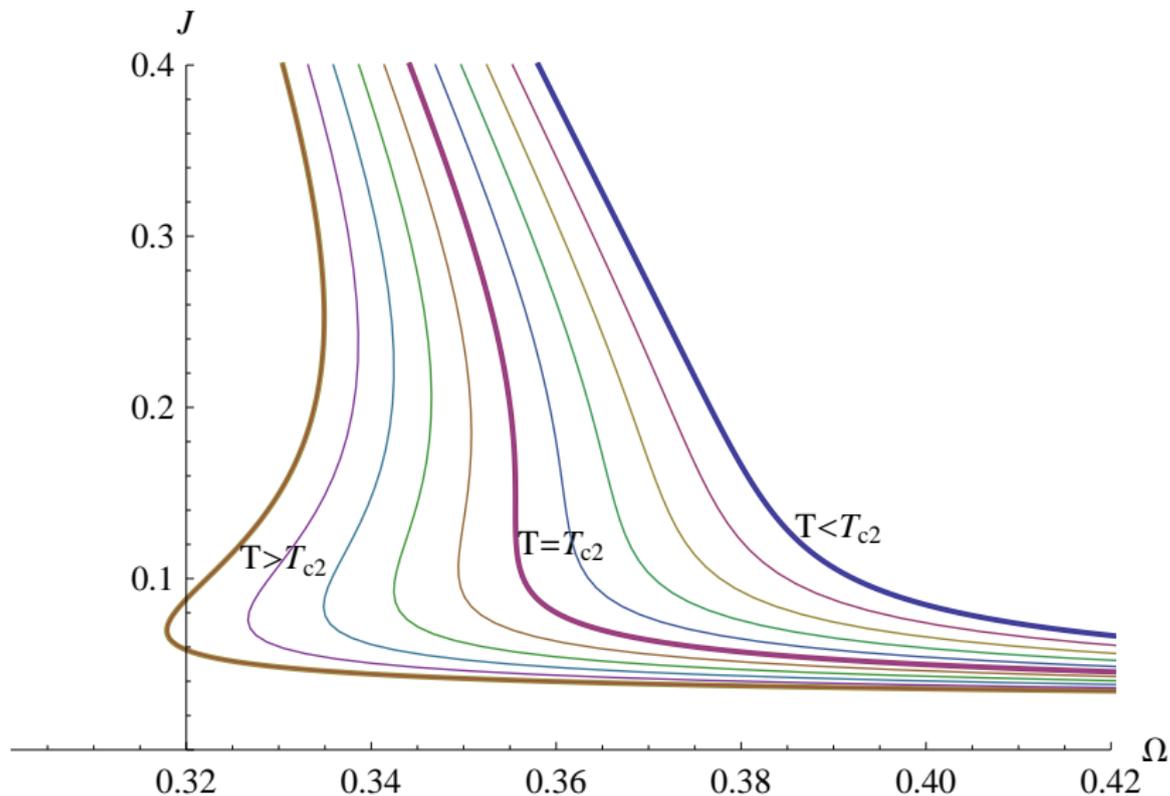
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- $\alpha = \alpha' = 0$
- $\gamma = \gamma' = 1$

# the second second order phase transition



The free energy  $F$  near the critical point:

$$F_s(\epsilon, \omega) = c_\epsilon \epsilon^2 + c_\omega \omega^{4/3}.$$

Thus we have the scaling symmetry of the free energy near the critical point,

$$F_s(\Lambda^{1/2}\epsilon, \Lambda^{3/4}\omega) = \Lambda F_s(\epsilon, \omega),$$

with all the scaling symmetry follows.

Write the critical exponents in terms of  $p$  and  $q$  as

$$\begin{aligned}\alpha &= \frac{2p - 1}{p}, \\ \beta &= \frac{1 - q}{p}, \\ \gamma &= \frac{2q - 1}{p}, \\ \delta &= \frac{q}{1 - q}.\end{aligned}\tag{10}$$

From the equations above, the critical exponents satisfy the following relations,

$$\begin{aligned}\alpha + 2\beta + \gamma &= 2 , \\ \alpha + \beta(\delta + 1) &= 2 , \\ \gamma(\delta + 1) &= (2 - \alpha)(\delta - 1) , \\ \gamma &= \beta(\delta - 1).\end{aligned}\tag{11}$$

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- Mean field gravity?

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